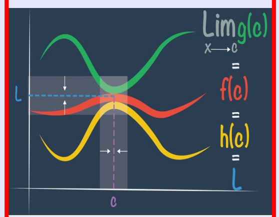
Calculus I

Lecture 30



Feb 19-8:47 AM

Use $\underbrace{\text{linear approximation}}$ to estimate $\sqrt[3]{1001}$.

$\sqrt[3]{1001} \approx \sqrt[3]{1000} = 10$

$f(x) \approx f(a) + f'(a)(x-a)$

$f(x) = \sqrt[3]{x}$

$a = 1000$

$f(x) = x^{1/3}$

$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

$f'(1000) = \frac{1}{3\sqrt[3]{1000^2}} = \frac{1}{3\cdot 10^2} = \frac{1}{300}$

$\sqrt[3]{x} \approx \sqrt[3]{1000} + \frac{1}{300}(x-1000)$

$\sqrt[3]{x} \approx 10 + \frac{1}{300}(x-1000)$

$\sqrt[3]{1001} \approx 10 + \frac{1}{300}(1001-1000)$

$= 10 + \frac{1}{300}\cdot 1 = \boxed{\frac{3001}{300}}$

$\boxed{10.00\overline{3}}$

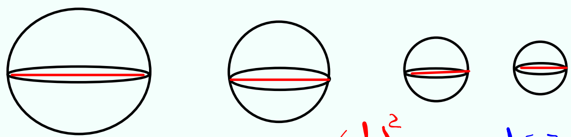
from Calc.

$\sqrt[3]{1001} \approx \boxed{10.00333}22$

Oct 21-7:27 AM

Snowball is melting so its surface area is decreasing at 1 cm^2/min.

Find the rate as which the diameter decreases when diameter is 10cm.



$V = \frac{4\pi r^3}{3}$

$S = 4\pi r^2 \rightarrow S = 4\pi\left(\frac{d}{2}\right)^2$

$S = \pi d^2$

$\frac{d}{dt}[d] = -$

$\frac{dS}{dt} = -1$

$d = 2r \qquad r = \frac{d}{2}$

$\frac{dS}{dt} = 2\pi d \frac{d}{dt}[d]$

$-1 = 2\pi \cdot 10 \cdot \frac{d}{dt}[d]$

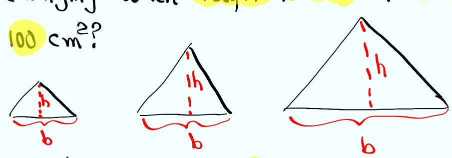
$\frac{d}{dt}[d] = \frac{-1}{20\pi}$ cm/min.

Oct 21-7:36 AM

The height of a triangle is increasing at a rate of 1 cm/min. $\frac{dh}{dt} = 1$ cm/min.

Its area is also increasing at a rate of 2 cm^2/min. $\frac{dA}{dt} = 2 cm^2/min$

At what rate is the base of the triangle changing when height is 10cm & area is 100 cm^2?



$A = \frac{bh}{2} \qquad 2A = bh \qquad 2\cdot 100 = b\cdot 10 \qquad \boxed{b=20}$

$\frac{d}{dt}[2A] = \frac{d}{dt}[bh]$

$2\frac{dA}{dt} = \frac{db}{dt}\cdot h + b\cdot\frac{dh}{dt}$

$2\cdot 2 = \frac{db}{dt}\cdot 10 + 20\cdot 1$

$4 - 20 = 10\frac{db}{dt} \qquad \frac{db}{dt} = \frac{-16}{10}$

$\frac{db}{dt} = -1.6$ cm/min

Oct 21-7:46 AM

$f(x) = 2 + 3x^2 - x^3$

1) Find $f'(x)$. $\quad f'(x) = 6x - 3x^2$

2) Solve $f'(x) = 0$. $\quad 6x - 3x^2 = 0$

$3x(2 - x) = 0$

$x = 0 \quad x = 2$

3) Find $f(x)$ when $f'(x) = 0$.

$f(0) = 2 + 3(0)^2 - 0^3 = \boxed{2}$

$f(2) = 2 + 3(2)^2 - 2^3 = 2 + 12 - 8 = \boxed{6}$

4) Find $f''(x)$. $= \frac{d}{dx}\left[f'(x)\right] = \frac{d}{dx}\left[6x - 3x^2\right] = 6 - 6x$

5) Solve $f''(x) = 0$. $\quad 6 - 6x = 0 \quad \boxed{x = 1}$

6) Find $f(x)$ when $f''(x) = 0$.

$f(1) = 2 + 3(1)^2 - 1^3 = 2 + 3 - 1 = \boxed{4}$

Oct 21-7:57 AM

$f(x) = \dfrac{x}{x-1}$

1) Domain $\quad x - 1 \neq 0 \quad x \neq 1 \qquad (-\infty, 1) \cup (1, \infty)$

V.A. $\quad x = 1$

2) All intercepts

$\boxed{\begin{array}{c} x\text{-Int.} \\ (0,0) \\ y\text{-Int.} \end{array}}$

$y = 0 \quad f(x) = 0 \quad \frac{x}{x-1} = 0 \quad \boxed{x = 0}$

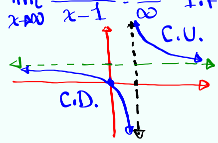
$x = 0 \quad f(0) = \frac{0}{0-1} = 0$

3) Find $\lim_{x \to \infty} f(x)$

$\lim_{x \to \infty} \frac{x}{x-1} = \frac{\infty}{\infty}$ I.F. $\quad \lim_{x \to \infty} \frac{\frac{x}{x}}{\frac{x}{x} - \frac{1}{x}} = \lim_{x \to \infty} \frac{1}{1 - \frac{1}{x}}$ $\quad \frac{1}{x} \to 0$

$= 1$

H.A.



$f(x) = \frac{x}{x-1}$

$f'(x) = \frac{1(x-1) - x \cdot 1}{(x-1)^2} = \frac{-1}{(x-1)^2}$

$f'(x)$ is undefined at $x = 1$

$f'(x) = -(x-1)^{-2}$

$\boxed{\begin{array}{l} f'(x) < 0 \\ f(x) \text{ is decreasing} \end{array}}$

$f''(x) = -(-2)(x-1)^{-3} \cdot 1$

$f''(x) = \frac{2}{(x-1)^3}$

$f''(x)$ is undefined at $x = 1$

If $x > 1$, $f''(x) > 0 \to$ Concave up

If $x < 1$, $f''(x) < 0 \to$ Concave down

Oct 21-8:04 AM

$f(x) = \dfrac{x-1}{x^2}$

1) Domain $x \neq 0$ $(-\infty, 0) \cup (0, \infty)$ V.A. $x = 0$

2) All intercepts $\boxed{\begin{array}{c} x\text{-Int} \\ (1,0) \\ y\text{-Int} \\ \text{None} \end{array}}$ $y = 0$ $f(x) = 0$ $\dfrac{x-1}{x^2} = 0$

$x-1 = 0$

$\boxed{x = 1}$

$x = 0$

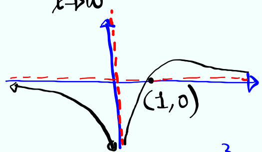
$x \neq 0$

3) $\lim\limits_{x \to \infty} f(x)$

$\lim\limits_{x \to \infty} \dfrac{x-1}{x^2} = \dfrac{\infty}{\infty}$

$\lim\limits_{x \to \infty} \dfrac{\frac{x}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2}} = \lim\limits_{x \to \infty} \dfrac{\cancel{\frac{1}{x}} - \cancel{\frac{1}{x^2}}}{1} = 0$

H.A. $y = 0$



$f(x) = \dfrac{x-1}{x^2}$

$f(x) = \dfrac{x}{x^2} - \dfrac{1}{x^2}$

$f(x) = \dfrac{1}{x} - \dfrac{1}{x^2}$

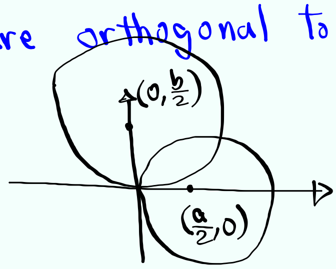
$f(x) = x^{-1} - x^{-2}$

$f'(x) = -1x^{-2} + 2x^{-3}$

$f''(x) = 2x^{-3} - 6x^{-4}$

Oct 21-8:17 AM

Show $x^2 + y^2 = ax$ & $x^2 + y^2 = by$ are orthogonal to each other.



$2x + 2y\dfrac{dy}{dx} = a$

$\boxed{\dfrac{dy}{dx} = \dfrac{a-2x}{2y}}$

$2x + 2y\dfrac{dy}{dx} = b\dfrac{dy}{dx}$

$\boxed{\dfrac{dy}{dx} = \dfrac{2x}{b-2y}}$

Product of derivatives $= -1$

$\dfrac{a-2x}{2y} \cdot \dfrac{2x}{b-2y} = \dfrac{2ax - 4x^2}{2by - 4y^2}$

$= \dfrac{2(x^2+y^2) - 4x^2}{2(x^2+y^2) - 4y^2} = \dfrac{2y^2 - 2x^2}{2x^2 - 2y^2} = \dfrac{y^2 - x^2}{x^2 - y^2} = -1$

Oct 21-8:28 AM